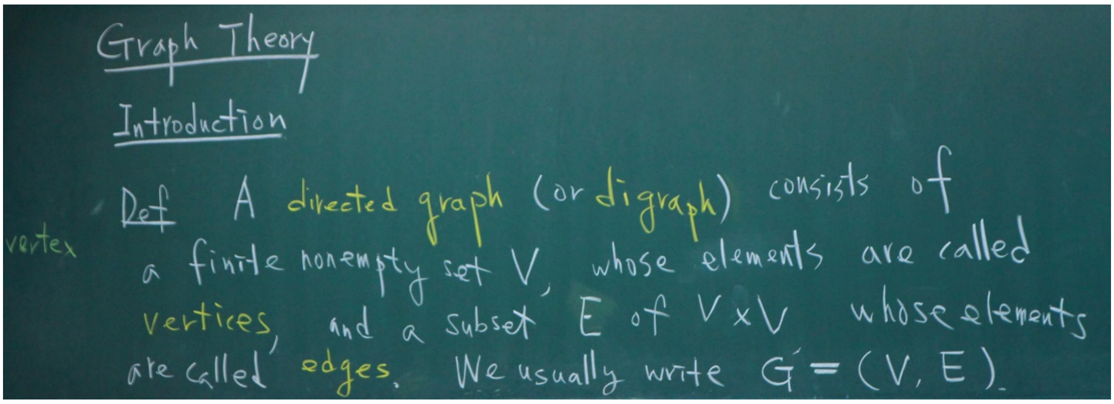
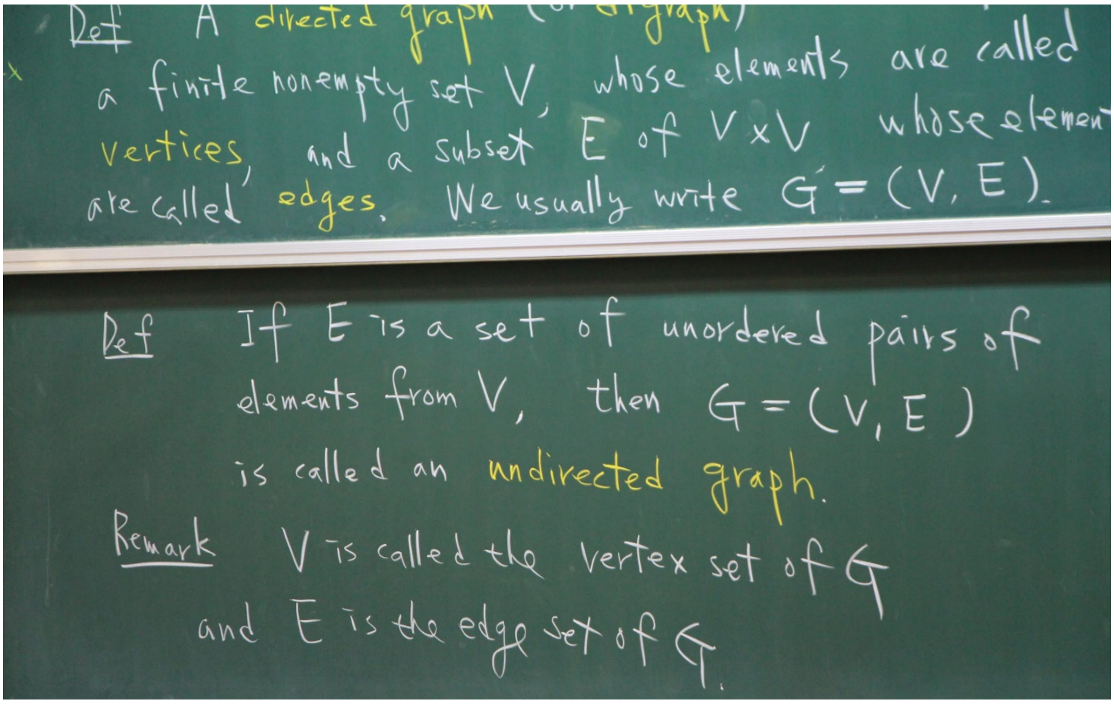
【10920 趙啟超教授離散數學 / 第 18 堂版書】





Graph Theory

Introduction

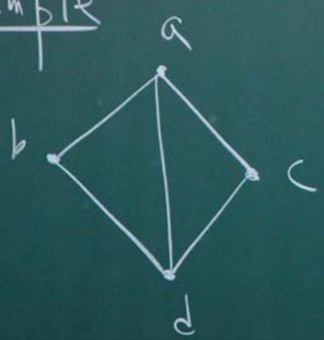
vertex

Def A directed graph (or digraph) consists of a finite nonempty set V, whose elements are called vertices, and a subset E of $V \times V$ whose elements are called edges. We usually write $G = (V, E)$.

Def If E is a set of unordered pairs of elements from V, then $G = (V, E)$ is called an undirected graph.

Remark V is called the vertex set of G and E is the edge set of G.

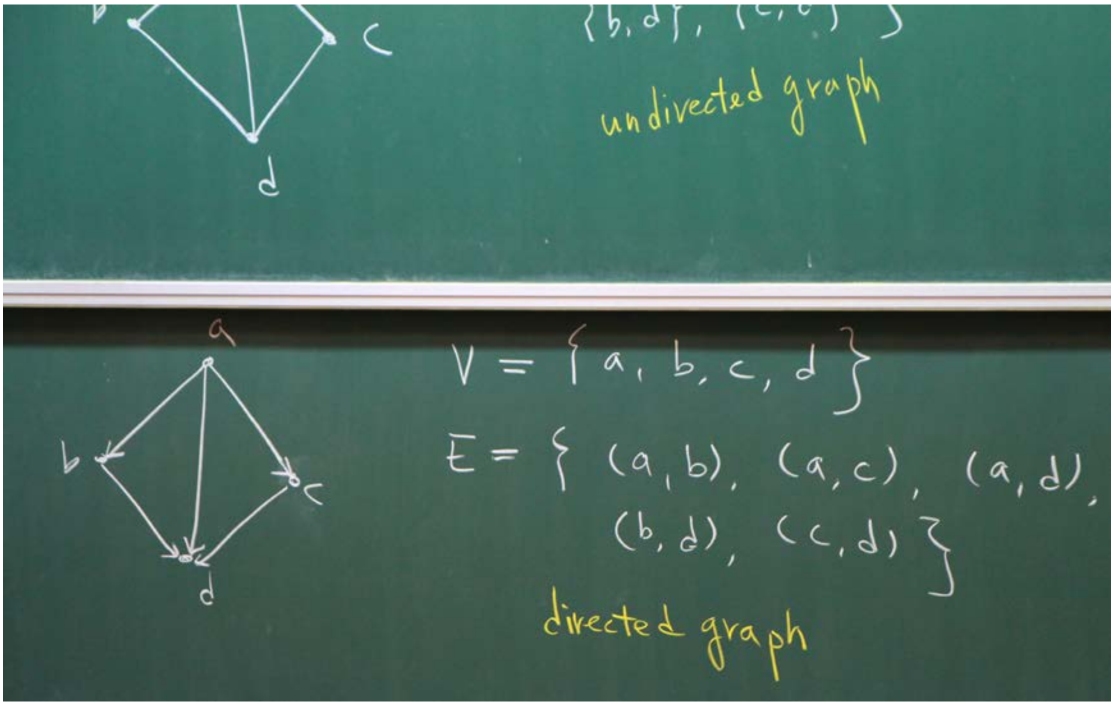
Example



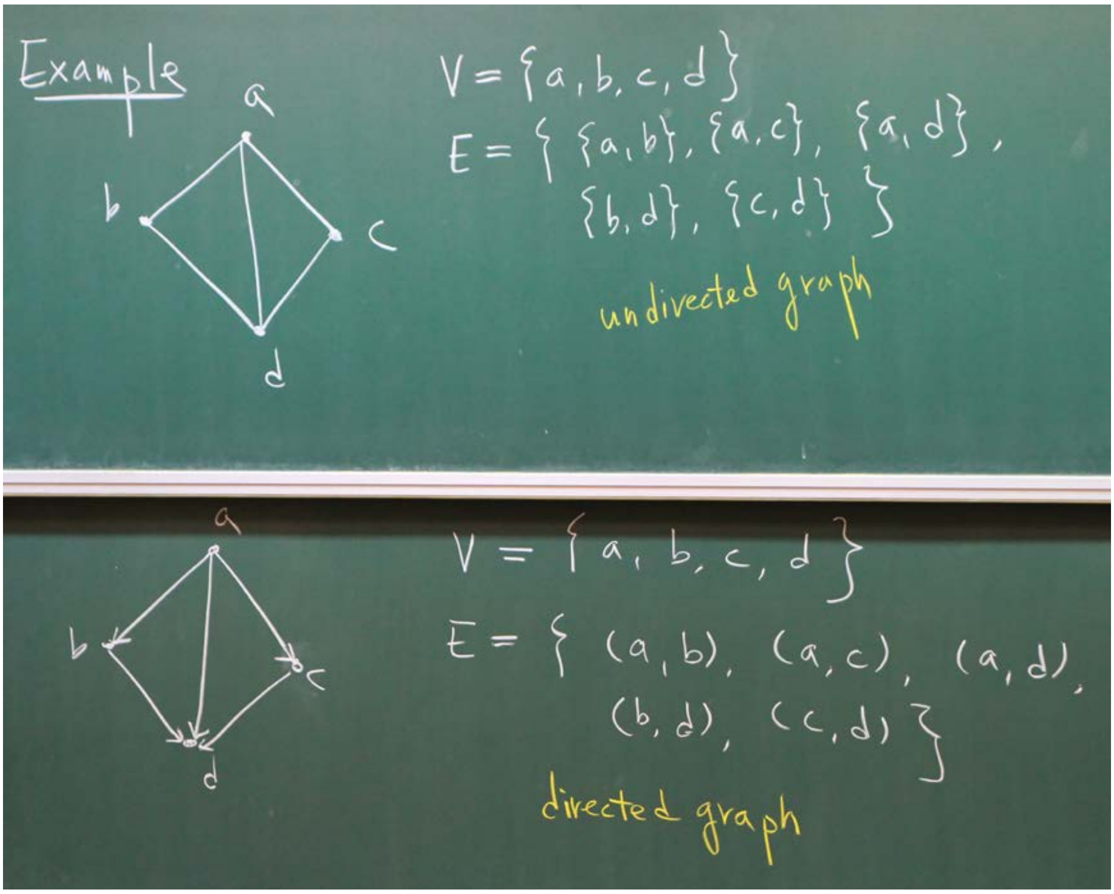
$V = \{a, b, c, d\}$

$E = \{ \{a,b\}, \{a,c\}, \{a,d\}, \{b,d\}, \{c,d\} \}$

undirected graph



c
d
undirected graph
a
b
c
d
V = {a, b, c, d}
E = { (a, b), (a, c), (a, d),
(b, d), (c, d) }
directed graph



Example
a
b
c
d
V = {a, b, c, d}
E = { {a, b}, {a, c}, {a, d},
{b, d}, {c, d} }
undirected graph
a
b
c
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V = {a, b, c, d}
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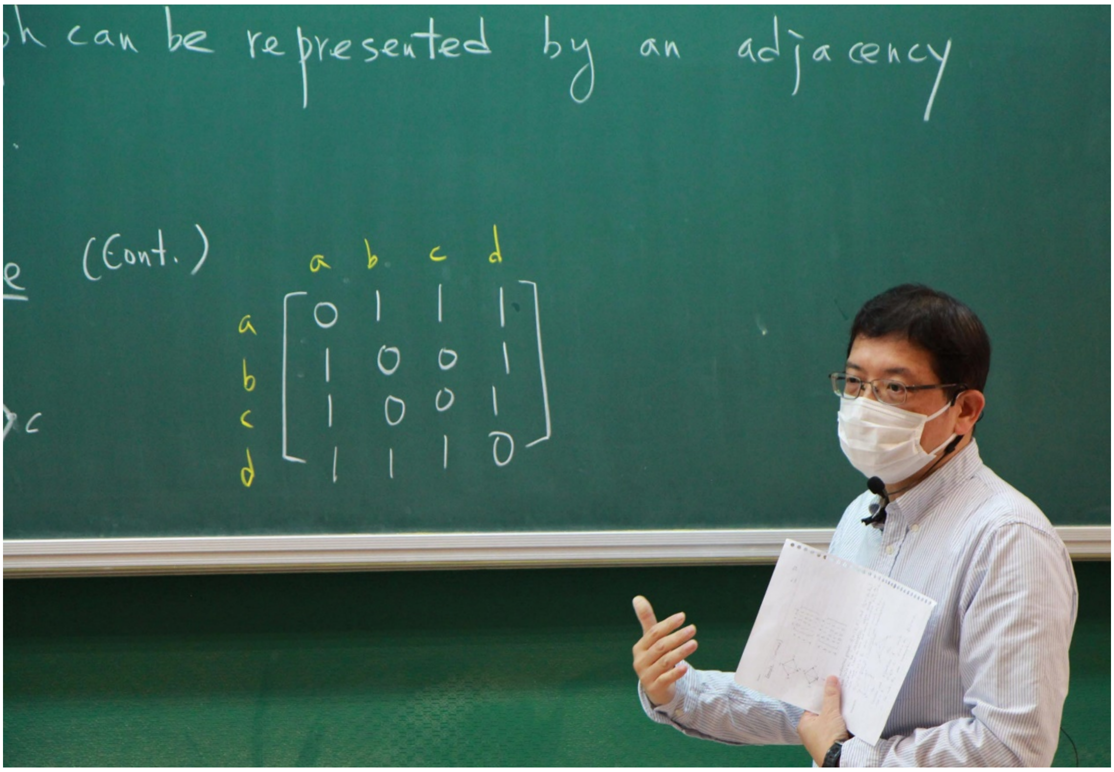
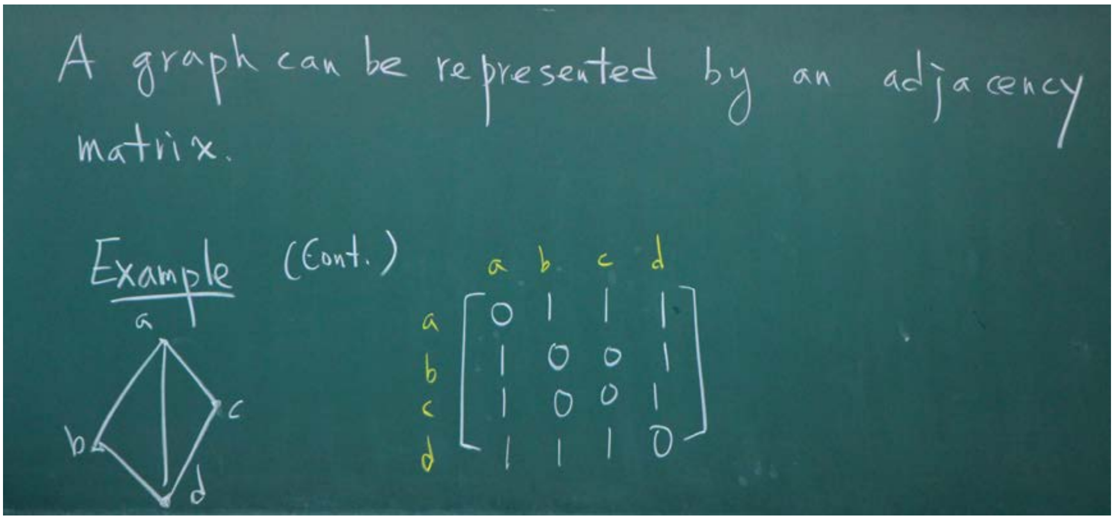
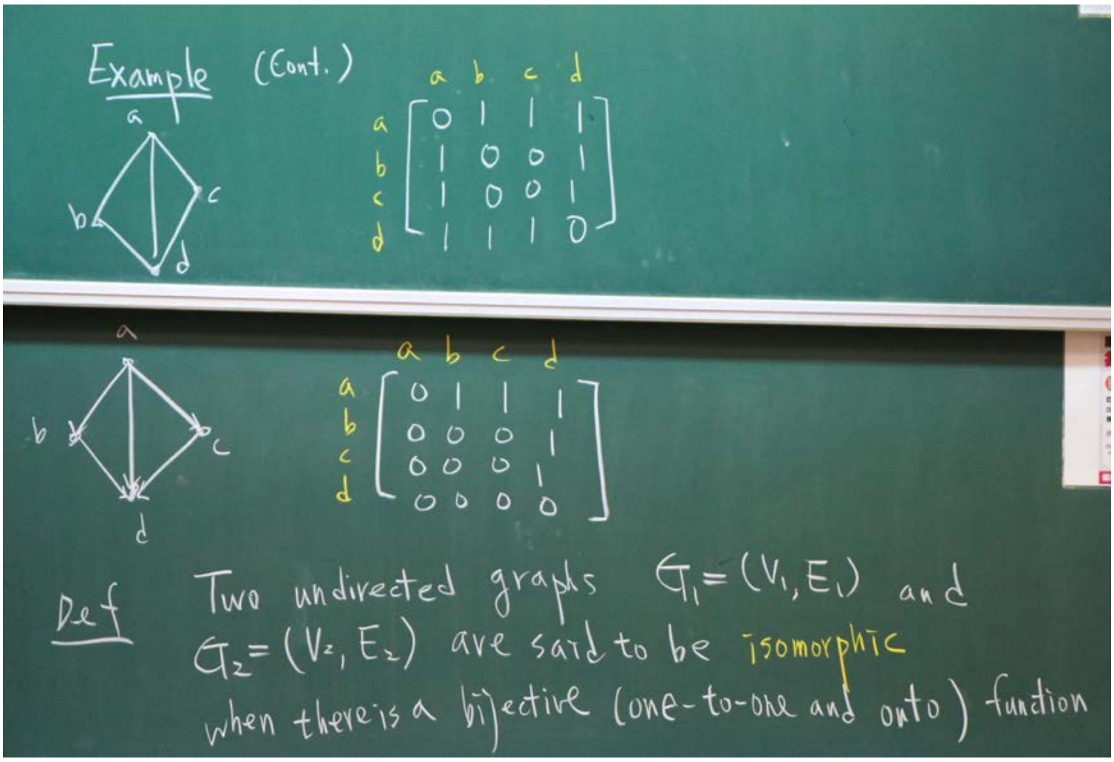
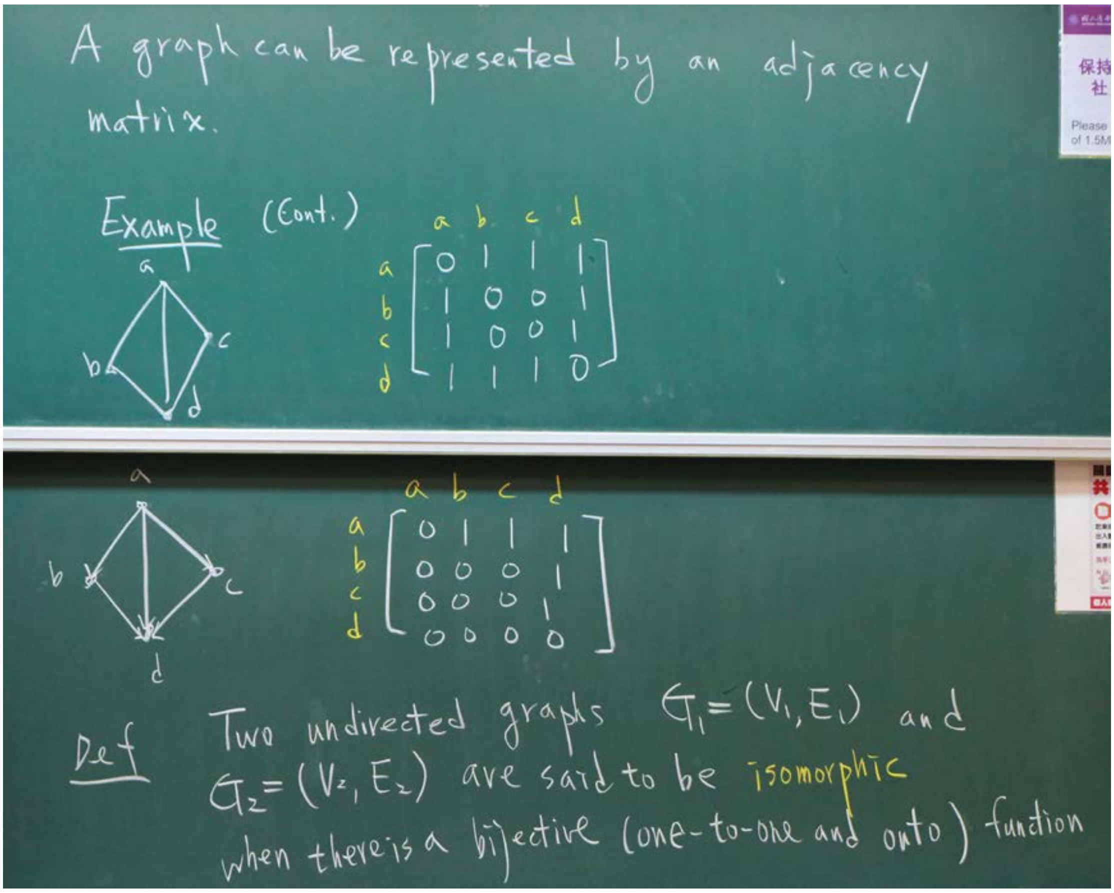


h can be represented by an adjacency
(Cont.)
a b c d
a 0 1 1 1
b 1 0 0 1
c 1 0 0 1
d 1 1 1 0



A graph can be represented by an adjacency matrix.
Example (Cont.)
a
b
c
d
a b c d
a 0 1 1 1
b 1 0 0 1
c 1 0 0 1
d 1 1 1 0


Example (Cont.)
a b c d
a 0 1 1 1
b 1 0 0 1
c 1 0 0 1
d 1 1 1 0
a b c d
a 0 1 1 1
b 0 0 0 1
c 0 0 0 1
d 0 0 0 0
Def Two undirected graphs G1=(V1,E1) and
G2=(V2,E2) are said to be isomorphic
when there is a bijective (one-to-one and onto) function


A graph can be represented by an adjacency matrix.
Example (Cont.)
a b c d
a 0 1 1 1
b 1 0 0 1
c 1 0 0 1
d 1 1 1 0
a b c d
a 0 1 1 1
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c 0 0 0 1
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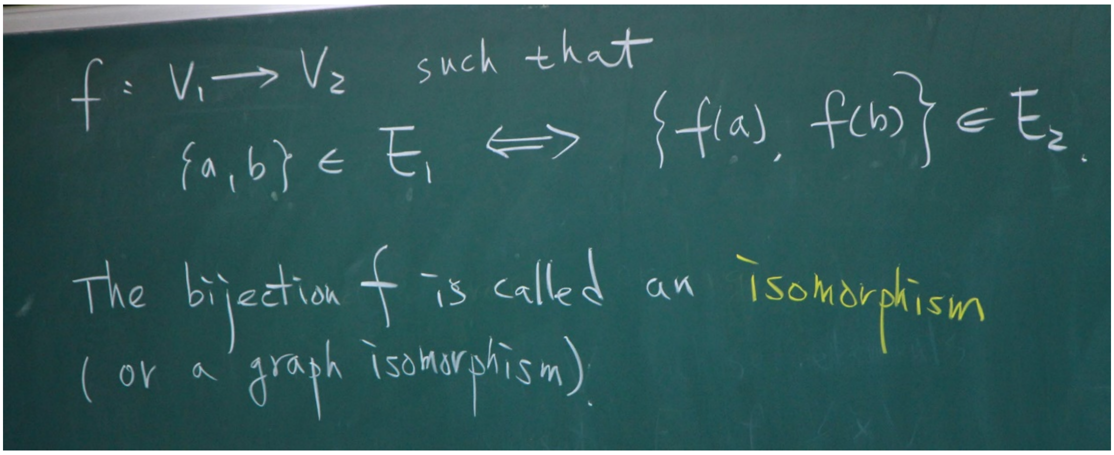
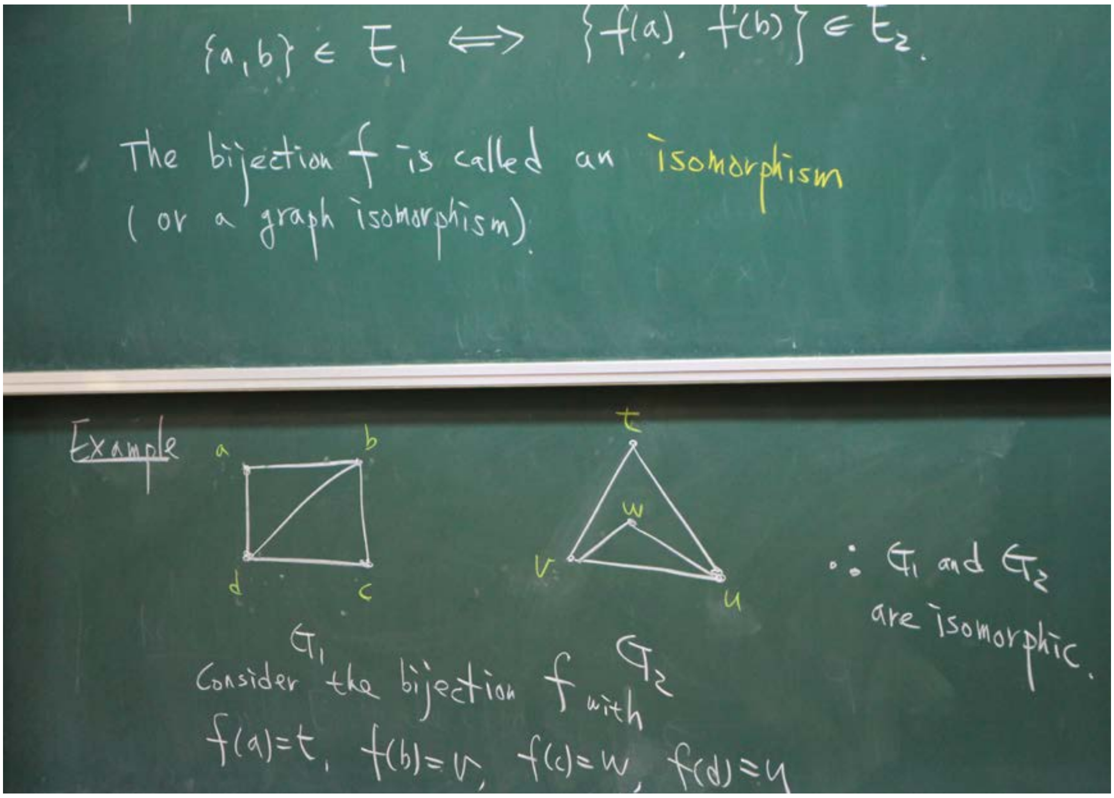

f : V1 → V2 such that
{a, b} ∈ E1 ⟺ {f(a), f(b)} ∈ E2.
The bijection f is called an isomorphism
(or a graph isomorphism).

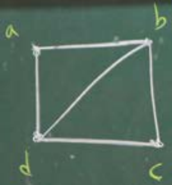

{a, b} ∈ E1 ⟺ {f(a), f(b)} ∈ E2.
The bijection f is called an isomorphism
(or a graph isomorphism).
Example
a
b
d
c
G1
t
w
v
u
G2
Consider the bijection f with
f(a)=t, f(b)=v, f(c)=w, f(d)=u
∴ G1 and G2 are isomorphic.

$f : V_1 \to V_2$ such that

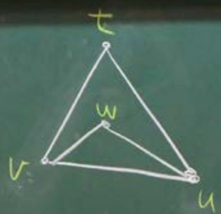
$$\{a,b\} \in E_1 \iff \{f(a), f(b)\} \in E_2.$$

The bijection f is called an **isomorphism** (or a graph isomorphism).

Example



G_1



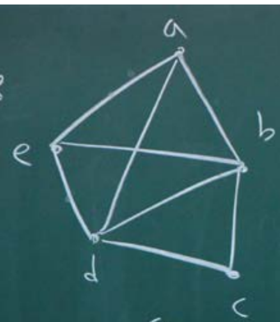
G_2

Consider the bijection f with

$f(a)=t$, $f(b)=v$, $f(c)=w$, $f(d)=u$

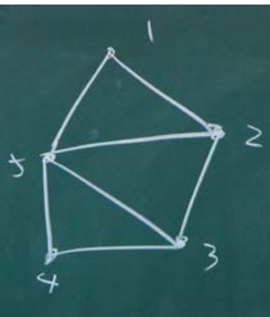
$\therefore$ G_1 and G_2 are isomorphic.

Example



G_1

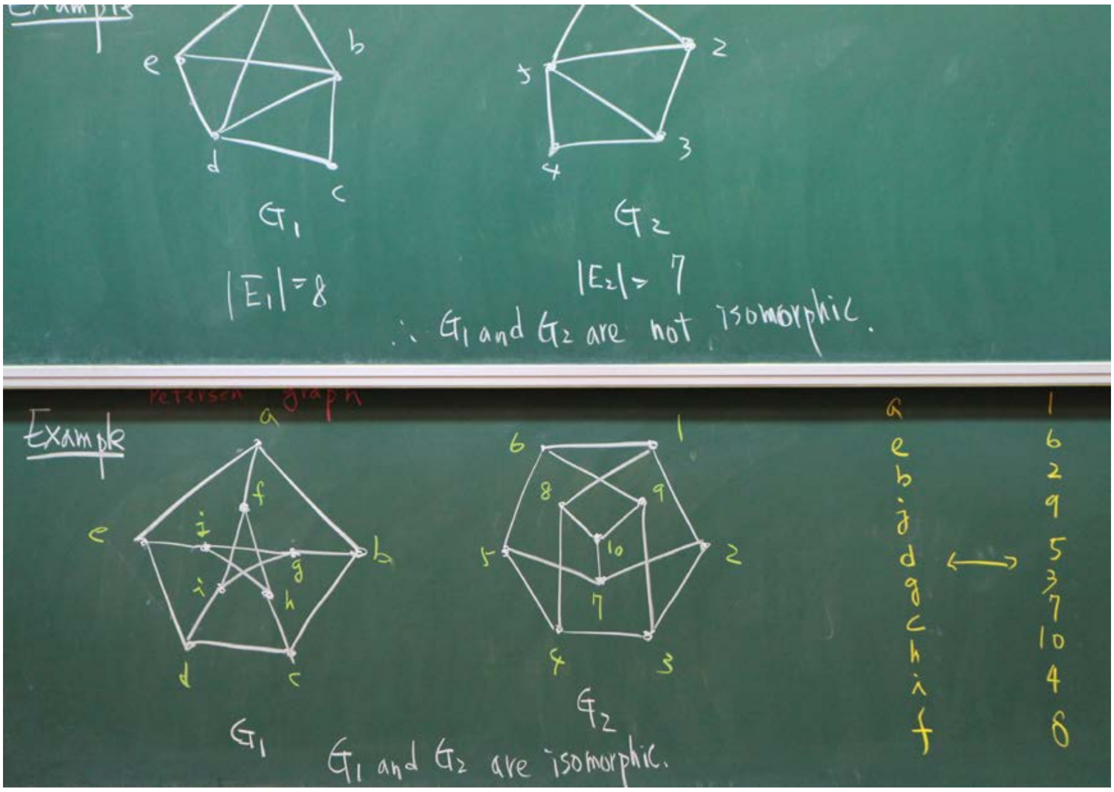
$|E_1| = 8$

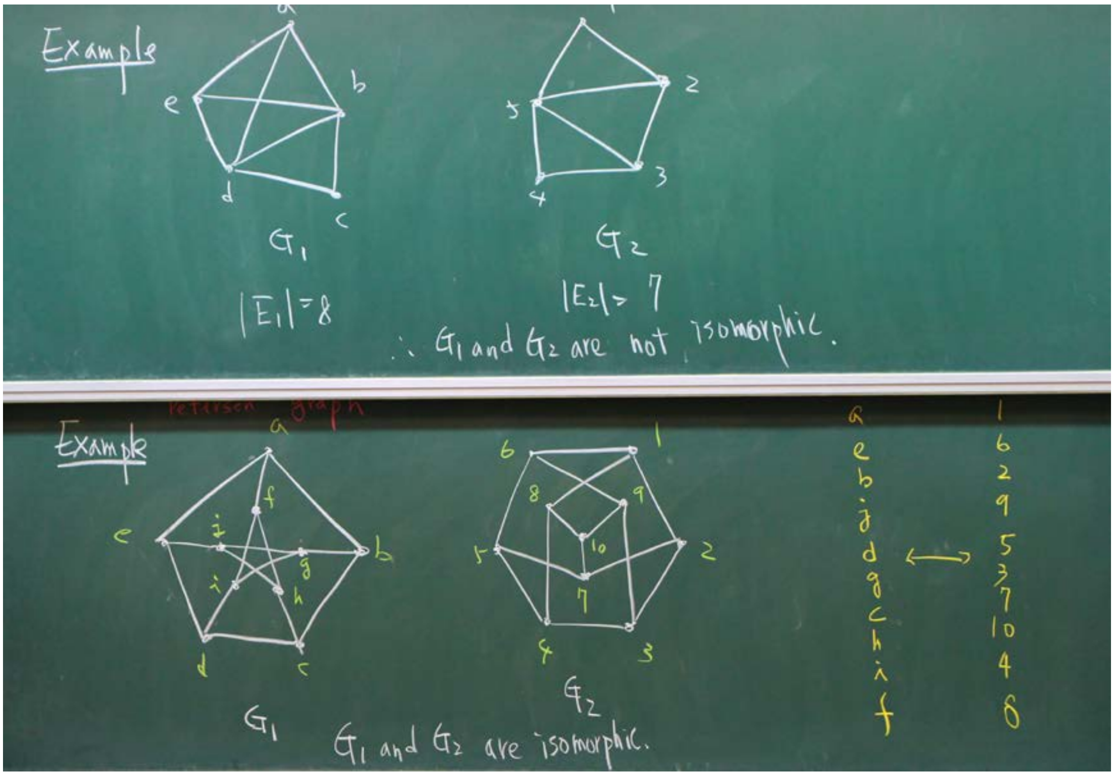


G_2

$|E_2| = 7$

$\therefore$ G_1 and G_2 are not isomorphic.


Example
e
b
d
c
5
2
4
3
G1
G2
|E1|=8
|E2|=7
∴ G1 and G2 are not isomorphic.
Petersen graph
Example
G1
G2
G1 and G2 are isomorphic.
a e b j d g c h i f
1 6 2 9 5 3 7 10 4 8


Example
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